A New Sum-Rate Outer Bound for Interference Channels with Three Source-Destination Pairs

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Abstract—**This paper derives a novel sum-rate outer bound for the general memoryless interference channel with three users. The derivation is a generalization of the techniques developed by Kramer and by Etkin *et al* for the Gaussian two-user channel. For the Gaussian channel the proposed sum-rate outer bound outperforms known bounds for certain channel parameters.**

Index Terms—**Interference channel; Outer bound; Sum-capacity.**

I. INTRODUCTION

An interference channel models an ad-hoc wireless network where several uncoordinated source-destination pairs share the same channel thereby creating undesired mutual interference at the receivers. Today's networks are designed to avoid interference through resource division among users because interference is considered the bottleneck of high-speed data networks. It is well known however that user orthogonalization, in frequency, time, space or code domain, is in general suboptimal in terms of performance. With advances in computing technology, it has become possible to design communication strategies to *manage* the interference. This trend has renewed the interest in the ultimate limits of interference networks. Much progress has been made in the past few years on understanding the capacity of the Gaussian interference channel with two source-destination pairs. However, interference channels with more than two source-destination pairs, or non-Gaussian channels, are far less understood. The objective of this work is to investigate the maximum throughput, or sum-rate, or sum-capacity, of the general memoryless interference channel with three source-destination pairs. The generalization of the proposed bounding technique to the whole capacity region and to an arbitrary number of source-destination pairs is presented in [1].

Before revising past work on interference networks and outlining our main contributions, we formally introduce the network problem considered in this paper.

A. Problem Definition

Our notation follows the convention in [2]. The channel considered in this work is depicted in Fig. 1. An InterFerence Channel with three source-destination pairs (3-IFC) is a multi-terminal network where source i, $i \in \{1, 2, 3\}$, wishes to communicate to destination i through a shared memoryless channel with transition probability $P_{Y_1, Y_2, Y_3 | X_1, X_2, X_3}$. Each source i, $i \in \{1, 2, 3\}$, encodes an independent message W_i
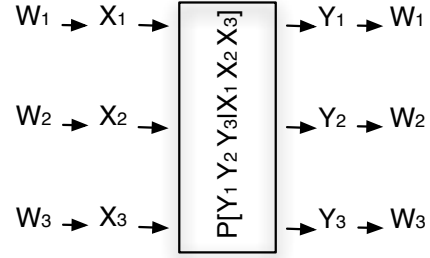

Fig. 1. The general memoryless InterFerence channel with three source-destination pairs (3-IFC) considered in this work.

of rate $R_i \in \mathbb{R}_+$ into a codeword of length $n \in \mathbb{N}$. We adopt standard definitions of codes, achievable rates and capacity region [2], that is, the capacity region is the convex closure of the set of rate-triplet (R_1, R_2, R_3) for which the error probability goes to zero as the block-length $n \to \infty$. As for other channels without destination cooperation, the capacity region of the 3-IFC only depends on the channel marginals $P_{Y_k | X_1, X_2, X_3}$, $k \in \{1, 2, 3\}$, and not on the whole joint channel transition probability $P_{Y_1, Y_2, Y_3 | X_1, X_2, X_3}$.

B. Past Work

The capacity region of a general memoryless 3-IFC is not known – not even the capacity of the 2-IFC has been characterized in full generality at present.

For a 2-IFC, the capacity region is known if: a1) the interfering signal is strong at each destination [3], [4]; if the interference is very strong, then interference does not reduce capacity [5]; b1) the outputs are deterministic functions of the inputs and invertible given the intended signal [6]; and c1) the channel has a special form of degradeness [3], [7], [8]. The largest known achievable region, due to Han and Kobayashi (HK) [9], uses rate-splitting and simultaneous decoding of the intended message and part of the interfering message. The best outer bound without auxiliary random variables is due to Sato [10], and with auxiliary random variables is due to Carleial [11] (see also Kramer [12, Th.5]).

For the Gaussian 2-IFC, the capacity region is fully known in strong interference only [5], [13], [14], that is, when the interference is strong at each destination. The sum-capacity is however known in the following cases: a2) in mixed

interference [15], [16], that is, when one interfering signal is strong and the other is weak; b2) for the Z-channel [17], that is, when only one destination experiences interference; and c2) in very weak interference at both destinations [16], [18], [19]. In the mixed and weak interference regimes, a simple message-splitting in the HK region is to within two bits [20] of the outer bound proposed in [21] for all channel parameters. The best outer bound for the Gaussian 2-IFC is obtain by intersecting the regions derived by Kramer in [12, Th.1] and in [12, Th.2], by Etkin *et al.* in [21], and the region independently obtained in [16], [18], [19] and later further tighten by Etkin in [22].

Very few results are available for a general memoryless K-IFC with $K \geq 3$. General inner bound regions are lacking. A straightforward generalization of the HK approach, whereby each user has a different (sub)message for every subset of non-intended receivers, would require the specification of $K2^{K-1}$ (sub)rates. The resulting region would have $K2^{(K+1)2^{K-2}-1}$ bounds and would still require an application of the Fourier-Motzkin elimination procedure in order to be expressed as a function of K rates only. Thus the HK approach for more than two users appears impractical because of its super-exponential complexity in the number of users. The HK approach might also be suboptimal in general. In fact, decoding at each receiver in a K-IFC is impaired by the joint effect of all the interferers, rather by each interferer separately. Consequently, coding schemes that deal directly with the effect of the combined interference could have superior performance in terms of achievable rates than the HK approach. Examples of such coding schemes for the Gaussian K-IFC are *interference alignment* [23] and *structured codes* [24]–[26].

In Gaussian noise, channels with special structure have been investigated: a3) the "fully symmetric" 3-IFC, whereby all interfering links have the same strength and all direct links have the same strength, was considered in [27]; a genie-aided outer bound that provides a group of receivers with sufficient side information so that they can decode a subset of the users as in a Multiple Access Channel (MAC) channel was also discussed in [27] and was later generalized in [28] to any number of users and any general channel matrix structure (however the resulting outer bound appears very difficult to evaluate in closed form); b3) the "cyclic symmetric" channel, whereby all receivers have a statistically equivalent output up to cyclic permutation of the user indices and are interfered by one other user only, was considered in [29]; it was shown that a generalization of the approach of [21, Th.1] gives capacity to within two bits when the interference is weak; if instead the interference is strong, the whole capacity region is given by an application of [12, Th.1] to each receiver; c3) the high-SNR linear deterministic approximation of the "cyclic symmetric" 3-IFC, without the restriction of having one-sided interference as in [29], was studied in [30]; the sum-capacity was characterized for almost all choices of parameters for the case where one interferer is strong and the other is weak; the corresponding finite-SNR model was not discussed; d3) the "cyclic mixed strong-very strong" 3-IFC was studied in [31]; here again the whole capacity region is obtained

by applying [12, Th.1] to each receiver, assuming that each receiver $k \in \{1, 2, 3\}$ experiences strong interference from user $k-1$ and very strong interference from user $k+1$ (indices are defined modulus 3); the conditions given in [31] for the achievability of the outer bound are sufficient; e3) the one-to-many (only one source creates interference) and the many-to-one (only one destination experiences interference) channels were studied in [25]; in both cases capacity was determined up to a constant number of bits that is an increasing function of the number of users; the central contribution is to show that purely random codes (according to the definition in [24]), like in the HK scheme, fail to achieve the outer bound to within a constant gap; instead structured lattice codes are necessary to establish capacity to within a finite number of bits; as mentioned before, structured codes are well suited for multi-interferer problems because they deal with the aggregate interference seen at a destination; in particular, with lattice codes, each destination has to decode one "virtual" interferer no matter how many users are present in the network; f3) continuing on the advantages of structured codes, it is known that the notion of strong interference does not extend to $K \geq 3$ users in a straightforward manner [32] and that structured codes outperform purely random codes; in particular, lattices allow for an "alignment" of the interference observed at each receiver and can achieve the interference-free capacity under a milder requirement on the channel matrix than random codes [32]; finally, g3) the Degrees of Freedom (DoF) of the K-IFC was considered in [26], [33], [34] and references therein; in general, random codes that generalize the two-layer coding schemes of HK to the K-user case are strictly outperformed by lattice codes [26]; "interference alignment" is known to achieve $K/2$ DoF for certain channels [23]; it is however known that the DoF is discontinuous at all fully connected, rational gain matrices [34]; this points out that high-SNR analysis in problems with many parameters (like the K-IFC) is very sensitive to the way the different parameters are let grow to infinity; the generalized DoF analysis [21] appears more appropriate but its complexity is quadratic in the number of users; the generalized DoF of the fully symmetric K-IFC for any $K \geq 2$ is the same as that the 2-IFC except when all channel outputs are statistically equivalent [33] (in which case time division is optimal).

C. Contributions and Paper Organization

The central contribution of this paper is to propose a *framework to derive sum-rate outer bounds for the 3-IFC that naturally generalizes to the whole capacity region of any memoryless IFC with an arbitrary number of users* [1]. Our contributions are as follows: 1) In Section II we derive a sum-rate outer bound for the *general memoryless 3-IFC* that generalizes the techniques originally developed by Kramer [12] and by Etkin *et al* [21] for the Gaussian 2-IFC; 2) In Section III we evaluate the bound derived in Section II for the Gaussian channel. We show that *the proposed bound improves on existing bounds for certain channel parameters.* Section IV concludes the paper.

II. MAIN RESULT FOR THE GENERAL 3-IFC

We divide the presentation of our novel sum-rate outer bound into two parts: Th.1 generalizes the approach of Kramer [12, Th.1] and Th.2 generalizes the approach of Etkin *et al* [21, Th.1]. Our proposed outer bound in the intersection of the regions in Th.1 and Th.2.

Theorem 1. *The sum-rate of a general memoryless 3-IFC is upper bounded by:*

$$
\begin{aligned}
R_1 + R_2 + R_3 &\leq I(Y_1; X_1, X_2, X_3, Q) \\
&\quad + I(Y_2; X_2, X_3 | X_1, Y_1, Q) \\
&\quad + I(Y_3; X_3 | X_1, Y_1, X_2, Y_2, Q),
\end{aligned} \tag{1}
$$

for some input distribution $P_{X_1, X_2, X_3, Q} = P_Q \prod_{k=1}^{3} P_{X_k | Q}$. By exchanging the role of the users in (1), other $(3! - 1) = 5$ sum-rate bounds can be obtained. Moreover, the sum-rate bound in (1) can be minimized with respect to the joint probability $P_{Y_1, X_3, Y_3 | X_1, X_3, X_3}$ as long as the marginal probabilities $P_{Y_k | X_1, X_2, X_3}$, $k \in \{1, 2, 3\}$, are preserved.

Proof: By Fano's inequality:

$$
n(R_1 + R_2 + R_3) \leq \sum_{k=1}^{3} I(W_k; Y_k^n)
$$

$$
\leq \sum_{k=1}^{3} I(W_k; Y_k^n, \ Y_{k-1}^n, W_{k-1}, \ldots, Y_1^n, W_1)
$$

$$
= H(Y_1^n) + H(Y_2^n | Y_1^n, W_1) + H(Y_3^n | Y_2^n, Y_1^n, W_1, W_2)
$$

$$
- H(Y_3^n, Y_2^n, Y_1^n | W_1, W_2, W_3).
$$

By continuing with standard inequalities (see [1] for details) the bound in (1) can be obtained. The joint channel transition probability can be optimized so as to tighten the sum-rate bound in (1), subject to preserving the marginals, because the capacity region only depends on the channel conditional marginal probabilities [35]. ∎

We remark that:

1) The proposed bound reduces to [12, Th.1] for the Gaussian 2-IFC when $\mathcal{X}_3 = \emptyset$ (see [12, eq.(34)]). Th.1 however holds for *any* memoryless IFC.

2) As described in [1], Th.1 can be extended to *any number of users K* and to *any partial sum-rate*, in which case the derived region contains $N(K) = \sum_{k=1}^{K} \binom{K}{k} k!$ bounds. For $K = 2$, the region has $N(2) = 4$ bounds as in [12, Th.1] (two single-rate bounds and two sum-rate bounds). For $K = 3$, the region has $N(3) = 15$ bounds, of which the $\binom{3}{3} 3! = 6$ sum-rate bounds cannot be derived by silencing one of the users and by applying [12, Th.1] to the resulting 2-IFC (see Section III) and are the novel contribution of Th.1.

3) Every mutual information term in Th.1 contains *all* the inputs (X_1, X_2, X_3) and no auxiliary random variable. This implies that the bound can be easily evaluated for many channels of interest, including the Gaussian channel (see Section III).

4) Th.1 can be easily extended to memoryless channels without receiver cooperation. For example, the 2-IFC with generalized feedback (a.k.a. source cooperation) [36] was studied in [37], [38] and the extension to any number of users is discussed in [1]. The 2-user cognitive channel was considered in [39] and the 2-IFC with a cognitive relay in [40].

Theorem 2. *The sum-rate of a general memoryless 3-IFC is upper bounded by:*

$$
R_1 + R_2 + R_3
$$

$$
\leq \sum_{k=1}^{3} H(Y_k | S_k, Q) - H(S_k | Y_k, X_1, X_2, X_3, Q), \tag{2}
$$

for some input distribution $P_{X_1, X_2, X_3, Q} = P_Q \prod_{k=1}^{3} P_{X_k | Q}$ and such that side information set $\{S_k, k \in \{1, 2, 3\}\}$ coincides with the set $\{Y_{\backslash k}, k \in \{1, 2, 3\}\}$, where $Y_{\backslash k} \sim Y_k | X_k$, i.e., $Y_{\backslash k}$ is statistically equivalent to the channel output at destination k from which the intended signal X_k has been removed. Moreover, the sum-rate bound in (2) can be minimized with respect to the joint probability $P_{Y_k, S_k | X_1, X_2, X_3}$, $k \in \{1, 2, 3\}$, as long as the conditional marginal distributions are preserved.

Proof: By Fano's inequality:

$$
n(R_1 + R_2 + R_3) \leq \sum_{k=1}^{3} I(W_k; Y_k^n) \leq \sum_{k=1}^{3} I(X_k^n; Y_k^n, S_k^n)
$$

$$
= \sum_{k=1}^{3} H(S_k^n) - H(Y_k^n | X_k^n) + H(Y_k^n | S_k^n) - H(S_k^n | X_k^n, Y_k^n).
$$

By assuming that $\sum_{k=1}^{3} H(S_k^n) \leq \sum_{k=1}^{3} H(Y_k^n | X_k^n)$ (which is the case when the side information set $\{S_k, k \in \{1, 2, 3\}\}$ coincides with the set $\{Y_{\backslash k} \sim Y_k | X_k, k \in \{1, 2, 3\}\}$) and by continuing with standard inequalities (see [1] for details) the bound in (2) can be obtained. ∎

We remark that:

1) The proposed bound reduces to [21, Th.1] for the Gaussian 2-IFC when $\mathcal{X}_3 = \emptyset$ by setting $S_1 = Y_{\backslash 2}$ and $S_2 = Y_{\backslash 1}$. Th.2 is however tighter than [21, Th.1] for the Gaussian 2-IFC because the correlation between the Gaussian noise of the channel output Y_j and the Gaussian noise of the side information $Y_{\backslash k}$, $(j, k) \in \{1, 2, 3\}^2$, can be optimized (see Section III).

2) Th.2 holds for *any* memoryless 3-IFC.

3) Th.2 can be extended to *any number of users K* and to *any partial sum-rate*; some of the bounds in the so-derived region cannot be obtained by simply silencing all but two users and then applying [21, Th.1] to the resuling 2-IFC and are the novel contribution of Th.2.

4) Extensions of Th.2 to other channel models are possible but appear more involved than those of Th.1 Such an extension has been presented in [38], [41] for the 2-IFC with generalized feedback and in [40] for the 2-IFC with a cognitive relay.
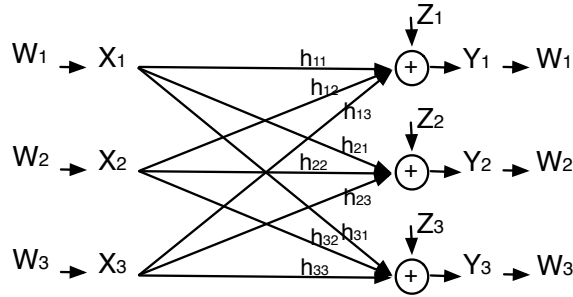
Fig. 2. The Gaussian 3-IFC considered in this work.

III. GAUSSIAN CHANNELS

The Gaussian channel model is introduced in Subsection III-A. A sum-rate outer bound derived form the results available for the Gaussian 2-IFC is described in Subsection III-B. Subsection III-C evaluates Th.1 and Th.2. Subsection III-D numerically compares the proposed sum-rate bounds with some of the results available in the literature and shows that there are channel parameters for which our proposed sum-rate bound is the tightest.

A. The Gaussian Channel Model

A SISO (single input single output) complex-valued Gaussian 3-IFC in *standard* form, depicted in Fig. 2, has outputs:

$$Y_i = \sum_{k=1}^{3} h_{i,k} X_k + Z_i,$$

with input power constraint $\mathbb{E}[|X_i|^2] \leq 1$ and noise $Z_i \sim \mathcal{N}(0,1)$, $i \in \{1,2,3\}$. The correlation among the Gaussian noises is irrelevant since the capacity only depends on the marginal noise distributions. The channel gains are fixed for the whole transmission duration and are known to all terminals. Without loss of generality, the direct link gains $h_{i,i}$, $i \in \{1,2,3\}$, can be taken to be real-valued (because receiver i can compensate for the phase of $h_{i,i}$) and strictly positive (if $|h_{i,i}|^2 = 0$ then the SNR at receiver i is zero even in absence of interference, which implies that $R_i = 0$, i.e., $X_i = 0$ is optimal and the system has effectively one less user). The Gaussian 3-IFC model is completely specified by the 3×3 channel matrix $\boldsymbol{H} : [\boldsymbol{H}]_{i,j} = h_{i,j}$, $(i,j) \in \{1,2,3\} \times \{1,2,3\}$.

B. Known sum-rate bounds

Sum-rate bounds for the 3-IFC can be obtained from known outer bound regions for the 2-IFC as follows. By silencing one of the users, which is equivalent to give the input signal of that user as side information to the other two receivers, the channel effectively reduces to a 2-IFC to which known sum-rate outer bounds apply. In particular, let:

$$r_k = \log(1 + |h_{kk}|^2), \ k \in \{1,2,3\},$$

and let r_{ij} be a sum-rate bound for a 2-IFC obtained by silencing all but user i and user j, $(i,j) \in \{1,2,3\} \times \{1,2,3\}$

with $i \neq j$. Then, the sum-rate can be upper bounded by:

$$r_{123}^{(\text{author})} = \min \Big\{ r_1 + r_2 + r_3, \quad r_1 + r_{23}, r_2 + r_{13}, r_3 + r_{12}, \\ \frac{r_{12} + r_{13} + r_{23}}{2} \Big\}, \tag{3}$$

where $\text{author} = \text{Kra}$ indicates that the 2-user rate bounds are obtained from [12, Th.1], i.e., for example:

$$r_{12}^{(\text{Kra})} = \min \Big\{ \log(1 + |h_{1,1}|^2 + |h_{1,2}|^2) + \Big[\log \Big(\frac{1 + |h_{2,2}|^2}{1 + |h_{1,2}|^2} \Big) \Big]^+, \\ \log(1 + |h_{2,1}|^2 + |h_{2,2}|^2) + \Big[\log \Big(\frac{1 + |h_{1,1}|^2}{1 + |h_{2,1}|^2} \Big) \Big]^+ \Big\},$$

and $\text{author} = \text{ETW}$ indicates that the 2-user sum-rate bounds are obtained from [21, Th.1], i.e., for example:

$$r_{12}^{(\text{ETW})} = \log \Big(1 + |h_{1,2}|^2 + \frac{|h_{1,1}|^2}{1 + |h_{2,1}|^2} \Big) \\ + \log \Big(1 + |h_{2,1}|^2 + \frac{|h_{2,2}|^2}{1 + |h_{1,2}|^2} \Big).$$

The bounds $r_{123}^{(\text{Kra})}$ and $r_{123}^{(\text{ETW})}$ will be compared to $r_{123}^{(\text{Th.1})}$ and $r_{123}^{(\text{Th.2})}$, the sum-rate from Th.1 and Th.2, respectively.

Remark 1 (Other known outer bounds for the 2-IFC and their generalization to the K-user case). *Outer bounds known in the litterature for the Gaussian 2-IFC, besides those in [12], [21], are [12, Th.2] (which is tighter than [21, Th.1] for some weak interference parameters) and [16], [18], [19], [22] (which is sum-rate optimal in very weak interference). It is left for future work to compare the r_{123} bounds computed according to (3) from these works with our Th.1 and Th.2.*

It is also left for future work to generalize the 2-IFC bounds in [12, Th.2] and in [16], [18], [19], [22] to the case of more than two users.

Remark 2 (Known outer bounds for some special K-IFC, $K \geq 3$). *As mentioned in the introduction, Gaussian K-IFC with special structure for the channel matrix $\boldsymbol{H}$ have been considered in the literature. In particular:*

1) *The sum-rate of the "cyclic mixed strong-very strong" 3-IFC [31] and of the "cyclic symmetric" 3-IFC in strong interference [29] is given by $r_{123}^{(\text{Kra})}$ in (3).*
2) *The capacity of the "cyclic symmetric" K-IFC in weak interference in [29] does not coincide with $r_{123}^{(\text{ETW})}$ in (3) but it is a special case of our Th.2.*
3) *It is left for future work to evaluate the MAC-based outer bound in [28] for the case of $K = 3$ users and compare it with our Th.1 and Th.2.*
4) *In our numerical examples we will also show the MAC sum-rate bound $r_{123}^{(\text{MAC})}$ obtained by letting all receivers cooperate so as to form a MAC channel with three single-antenna transmitters and a three-antenna receiver. The sum-capacity of this MAC channel is:*

$$r_{123}^{(\text{MAC})} = \min_{\boldsymbol{\Sigma}_{123}} \log \big(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{H}^H \boldsymbol{\Sigma}_{123}^{-1} \big), \tag{4}$$

where the minimization is over all positive-definite noise covariance matrix Σ_{123} constrained to have unit diagonal elements (i.e., "same conditional marginal" constraint, see (6)).

C. Evaluation of Th.1 and Th.2

Theorem 3. *For the Gaussian channel Th.1 reduces to:*

$$R_1 + R_2 + R_3$$
$$\leq I(Y_1; X_1) + \min_{\rho:|\rho|\leq 1}\Big\{I(Y_1, Y_2; X_2|X_1)$$
$$+ \max\big\{I(Y_1, Y_2; X_3|X_1, X_2), I(Y_3; X_3|X_1, X_2)\big\}\Big\}, \quad (5)$$

evaluated for iid $\mathcal{N}(0,1)$ inputs.

Proof: Since every mutual information term in Th.1 contains *all* inputs, the "Gaussian maximizes entropy" principle assures that jointly Gaussian inputs are optimal. Given the unitary power constraints, it is thus optimal to consider iid $\mathcal{N}(0,1)$ inputs in (1). Next we minimize the sum-rate with Gaussian inputs with respect to the noise covariance matrix:

$$\Sigma_{123} = \begin{pmatrix} 1 & \rho & \rho_1 \\ \rho^* & 1 & \rho_2 \\ \rho_1^* & \rho_2^* & 1 \end{pmatrix} \triangleq \begin{pmatrix} \Sigma_{12} & \boldsymbol{\rho} \\ \boldsymbol{\rho}^H & 1 \end{pmatrix}, \quad (6)$$

where $\boldsymbol{\rho} = (\rho_1, \rho_2)^T$ and Σ_{12} is the upper-left 2×2 principal submatrix of Σ_{123}.

We star by rewriting the sum-rate bound in (1) as:

$$R_1 + R_2 + R_3 \leq I(Y_1; X_1) + I(Y_1, Y_2; X_2|X_1)$$
$$+ I(Y_1, Y_2, Y_3; X_3|X_1, X_2). \quad (7)$$

By the non-negativity of mutual information, the last term in (7) can lower bounded by:

$$I(Y_1, Y_2, Y_3; X_3|X_1, X_2)$$
$$\geq \max\{I(Y_1, Y_2; X_3|X_1, X_2), I(Y_3; X_3|X_1, X_2)\}. \quad (8)$$

The lower bound in (8) is tight if we can show that Y_3 and (Y_1, Y_2) are one a degraded version of the other when conditioned on (X_1, X_2). Toward this goal, we whiten the noise in $(Y_1, Y_2)|_{(X_1, X_2)}$ and then perform maximal ratio combining so as to obtain an equivalent output:

$$Y_{\text{eq}} = \sqrt{\begin{pmatrix} h_{1,3}^* & h_{2,3}^* \end{pmatrix} \Sigma_{12}^{-1} \begin{pmatrix} h_{1,3} \\ h_{2,3} \end{pmatrix}} X_3 + Z_{\text{eq}}, \; Z_{\text{eq}} \sim \mathcal{N}(0,1).$$

Thus,

- CASE 1: if the SNR of Y_{eq} is larger than the SNR of $Y_3|_{(X_1, X_2)} \sim h_{33}X_3 + Z_3$, that is, if:

$$\begin{pmatrix} h_{1,3}^* & h_{2,3}^* \end{pmatrix} \Sigma_{12}^{-1} \begin{pmatrix} h_{1,3} \\ h_{2,3} \end{pmatrix} \geq |h_{33}|^2, \quad (9)$$

then $Y_3|_{(X_1, X_2)}$ is a degraded version of Y_{eq} and

$$I(Y_1, Y_2, Y_3; X_3|X_1, X_2) = I(Y_{\text{eq}}; X_3)$$
$$= I(Y_1, Y_2; X_3|X_1, X_2).$$

In this case, in order to determine the sum-rate in (5) we must still solve:

$$\min_{\rho:|\rho|\leq 1}\{I(Y_1, Y_2; X_2, X_3|X_1)\}, \quad (10)$$

where the minimization in (10) is subject to the constraint in (9). The optimal ρ in (10) *without* considering the constraint from (9) can be obtained by applying Lemma 5 in the Appendix with:

$$\boldsymbol{c}_1^T = (h_{1,2} \; h_{1,3}), \; \boldsymbol{c}_2^T = (h_{2,2} \; h_{2,3}),$$

to obtain that the optimal unconstrained ρ is $\rho^{(1)}$ with:

$$\rho^{(1)} = (t - \sqrt{t^2 - 1})e^{j\angle \boldsymbol{c}_1^H \boldsymbol{c}_2}, \quad (11)$$
$$t \triangleq \frac{(1 + \|\boldsymbol{c}_1\|^2)(1 + \|\boldsymbol{c}_2\|^2) - |\boldsymbol{c}_1^H \boldsymbol{c}_2|^2 - 1}{2|\boldsymbol{c}_1^H \boldsymbol{c}_2|} \geq 1.$$

The correlation coefficient $\rho^{(1)}$ can be the optimal solution for ρ in (5) under certain conditions that we will discuss later on.

- CASE 2: If the condition in (9) is not satisfied, then Y_{eq} is a degraded version of $Y_3|_{(X_1, X_2)}$ and

$$I(Y_1, Y_2, Y_3; X_3|X_1, X_2) = I(Y_3; X_3|X_1, X_2).$$

In this case, in order to determine the sum-rate in (5) we must still solve:

$$\min_{\rho:|\rho|\leq 1}\{I(Y_1, Y_2; X_2|X_1)\}, \quad (12)$$

where the minimization is subject to the complement condition of (9). The optimal ρ in (12) *without* considering the constraint from the complement condition of (9) can be obtained as follows. In $I(Y_1, Y_2; X_2|X_1)$ the signal X_3 acts as noise, hence, by rewriting (Y_1, Y_2) as:

$$Y_1' = \frac{Y_1}{\sqrt{1 + |h_{1,3}|^2}} = \frac{h_{1,2}}{\sqrt{1 + |h_{1,3}|^2}}X_2 + \frac{h_{1,3}X_3 + Z_1}{\sqrt{1 + |h_{1,3}|^2}},$$
$$Y_2' = \frac{Y_2}{\sqrt{1 + |h_{2,3}|^2}} = \frac{h_{2,2}}{\sqrt{1 + |h_{2,3}|^2}}X_2 + \frac{h_{2,3}X_3 + Z_2}{\sqrt{1 + |h_{2,3}|^2}},$$

we see that the correlation coefficient among the equivalent noises in Y_1' and Y_2' is:

$$\rho' \triangleq \frac{h_{1,3}h_{2,3}^* + \rho}{\sqrt{(1 + |h_{1,3}|^2)(1 + |h_{2,3}|^2)}}.$$

If the SNR of Y_1' is smaller than the SNR of Y_2', i.e.,:

$$\frac{|h_{1,2}|^2}{1 + |h_{1,3}|^2} \leq \frac{|h_{2,2}|^2}{1 + |h_{2,3}|^2} \quad (13)$$

then Y_1' can be made a degraded version of Y_2' if:

$$\rho' = \frac{h_{1,2}}{\sqrt{1 + |h_{1,3}|^2}}\frac{\sqrt{1 + |h_{2,3}|^2}}{h_{2,2}} \Longleftrightarrow$$
$$\rho^{(2a)} = \frac{h_{1,2}}{h_{2,2}}(1 + |h_{2,3}|^2) - h_{1,3}h_{2,3}^*. \quad (14)$$

If the condition in (13) is not satisfied, then Y_2' can be made a degraded version of Y_1' if:

$$\rho' = \frac{h_{2,2}}{\sqrt{1+|h_{2,3}|^2}} \frac{\sqrt{1+|h_{1,3}|^2}}{h_{1,2}} \Longleftrightarrow$$

$$\rho^{(2b)} = \frac{h_{2,2}}{h_{1,2}}(1+|h_{1,3}|^2) - h_{1,3}h_{2,3}^*. \quad (15)$$

The correlation coefficients $\rho^{(2a)}$ and $\rho^{(2b)}$ can be the optimal solution for ρ in (5) under certain conditions that we will discuss next.

The optimization over ρ in (5) can be carried out in closed form as follows (see for example [42, Sec. II.C]). If $\rho^{(1)}$ in (11) satisfies the condition in (9), then $\rho = \rho^{(1)}$ is optimal. If $\rho^{(2a)}$ in (14) satisfies the complement of the condition in (9), $|\rho^{(2a)}| \leq 1$, and the condition in (13), then $\rho = \rho^{(2a)}$ is optimal and the sum-rate in (5) becomes:

$$R_1 + R_2 + R_3 \leq \log\left(1 + \frac{|h_{1,1}|^2}{1+|h_{1,2}|^2+|h_{1,3}|^2}\right)$$

$$+ \log\left(1 + \frac{|h_{2,2}|^2}{1+|h_{2,3}|^2}\right) + \log\left(1+|h_{33}|^2\right). \quad (16)$$

If $\rho^{(2b)}$ in (15) satisfies the complement of the condition in (9) $|\rho^{(2b)}| \leq 1$, and the complement of the condition in (13), then $\rho = \rho^{(2b)}$ is optimal and the sum-rate in (5) becomes:

$$R_1 + R_2 + R_3 \leq \log\left(1 + \frac{|h_{1,1}|^2+|h_{1,2}|^2}{1+|h_{1,3}|^2}\right)$$

$$+ \log\left(1+|h_{33}|^2\right). \quad (17)$$

In all other cases, the optimal ρ in (5) is such that the condition in (9) holds with equality, that is, $\rho = \rho^{(3)}$ is optimal with:

$$\left|\rho^{(3)} - \frac{h_{1,3}h_{2,3}^*}{|h_{3,3}|^3}\right| = \sqrt{1 - \frac{|h_{1,3}|^2}{|h_{3,3}|^3}}\sqrt{1 - \frac{|h_{2,3}|^2}{|h_{3,3}|^3}}. \quad (18)$$

For cases 1 and 3 the closed-form expression for the sum-rate in (5) is quite involved and we do not explicitly write it here for sake of space. ∎

Theorem 4. *For the Gaussian channel Th.2 reduces to:*

$$R_1 + R_2 + R_3 \leq \min_{\pi}\left\{f_{k,\pi_k}\right\}, \quad (19)$$

where π is a permutation of the vector $(1,2,3)$ and where

$$f_{j,k} = \log\left(\frac{1+|\boldsymbol{r}_j\boldsymbol{r}_{\backslash k}^H|(q+\sqrt{q^2-1})}{1+\|\boldsymbol{r}_{\backslash k}\|^2}\right),$$

for $\boldsymbol{r}_k = (h_{k,1}, h_{k,2}, h_{k,3})$ (i.e., the set of channel coefficients seen at receiver k arranged in a row vector), with $[\boldsymbol{r}_{\backslash k}]_u = [\boldsymbol{r}_k]_u \delta_{k-u}$ (i.e., $\boldsymbol{r}_{\backslash k}$ equals $\boldsymbol{r}_k$ except for the k-th entry which is zero), and for $(j,k) \in \{1,2,3\}^2$

$$q = \frac{(1+\|\boldsymbol{r}_j\|^2)(1+\|\boldsymbol{r}_{\backslash k}\|^2) - |\boldsymbol{r}_j\boldsymbol{r}_{\backslash k}^H|^2 - 1}{2|\boldsymbol{r}_j\boldsymbol{r}_{\backslash k}^H|} \geq 1.$$

Proof: In the Gaussian case, for $k \in \{1,2,3\}$, we have:

$$Y_{\backslash k} \sim Y_k - h_{k,k}X_k$$

$$= \sum_{u \in \{1,2,3\}\backslash\{k\}} h_{k,u}X_u + Z_{\backslash k} = \boldsymbol{r}_{\backslash k}(X_1, X_2, X_3)^T + Z_{\backslash k},$$

where $Z_{\backslash k} \sim \mathcal{N}(0,1)$ whose correlation with the Gaussian noise on the channel output can be chosen so as to tighten the sum-rate bound in (2). Next, by applying Lemma 5 in the Appendix, we obtain:

$$\min_{\rho=\mathbb{E}[Z_j Z_{\backslash k}^*]:|\rho|\leq 1} h(Y_j|Y_{\backslash k}) - h(Z_{\backslash k}|Z_j)$$

$$= \min_{\rho=\mathbb{E}[Z_j Z_{\backslash k}^*]:|\rho|\leq 1} I(Y_j; X_1, X_2, X_3|Y_{\backslash k}) = f_{j,k},$$

for $f_{j,k}$ defined above. By considering all the permutations in (2) such that the side information set $\{S_k, k \in \{1,2,3\}\}$ coincides with the set $\{Y_{\backslash k}, k \in \{1,2,3\}\}$ we finally get the claimed expression for the sum-rate in (19). ∎

D. Numerical Example

In this section we compare our proposed sum-rate outer bound in Th.3 and in Th.4 with $r_{123}^{(\text{Kra})}$, $r_{123}^{(\text{ETW})}$ and $r_{123}^{(\text{MAC})}$ as defined in subsection III-B for the following class of channels:

$$\boldsymbol{H} = \begin{pmatrix} 1 & h_1 & h_2 \\ h_2 & 1 & h_1 \\ h_1 & h_2 & 1 \end{pmatrix}\sqrt{P}$$

The "fully symmetric" channel is obtained for $h_1 = h_2 = h$, the "cyclic symmetric" channel of [29] for $h_1 = h$, $h_2 = 0$, and the "mixed strong-very strong" channel of [31] for $|h_1|^2 \geq 1$, $|h_2|^2 \geq 1+|h_1|^2+1/P$. In Fig. 3 we plot the function:

$$\text{DoF}(P) \triangleq \min\{r_{123}/(r_1+r_2+r_3), 1\}$$

vs. $\alpha = \log(|h|^2 P)/\log(P)$ for $P = 20$dB. By the definition in (3), $r_{123}^{(\text{Kra})}/(r_1+r_2+r_3) \leq 1$ as well as $r_{123}^{(\text{ETW})}/(r_1+r_2+r_3) \leq 1$; however $r_{123}^{(\text{Th.1})}$ in (5), $r_{123}^{(\text{Th.2})}$ in (19) and $r_{123}^{(\text{MAC})}$ in (4) need not be smaller than the sum of the single-user rate bounds. The function $\text{DoF}(P)$ can be thought of as the DoF of the channel at a finite $\text{SNR} = P$; the DoF of the channel normalized by the number of users, as per definition commonly accepted in the literature, is $\text{DoF} = \lim_{P\to\infty}\text{DoF}(P)$.

The $\text{DoF}(P)$ of the "fully symmetric" channel is presented in Fig. 3(a). From [33], the $\text{DoF} = \lim_{P\to\infty}\text{DoF}(P)$ is as for the 2-user channel [21] for any $K \geq 2$ except at $\alpha = 1$ where it equals $1/K$ (because the channel matrix is always full rank for any $h \neq 1$ but is has unit rank for $h = 1$). From Fig. 3(a) we see that the DoF (yellow line) does not accurately predict performance at $P = 20$dB yet. For $h = 1$, that is when all channel outputs are statistically equivalent, the crude MAC bound (cyan line) is optimal. The new bound $r_{123}^{(\text{Th.1})}$ (solid blue line) is the tightest for approximately $\alpha \in [0.54, 1.15]$ (the range enlarges for smaller P). For approximately $\alpha \leq 0.54$ the bound $r_{123}^{(\text{ETW})}$ (dotted magenta line) is the tightest and for approximately $\alpha \geq 1.15$ the bound $r_{123}^{(\text{Kra})}$ (dotted blue line) is the tightest. The line labeled with "Others" (red line) refers
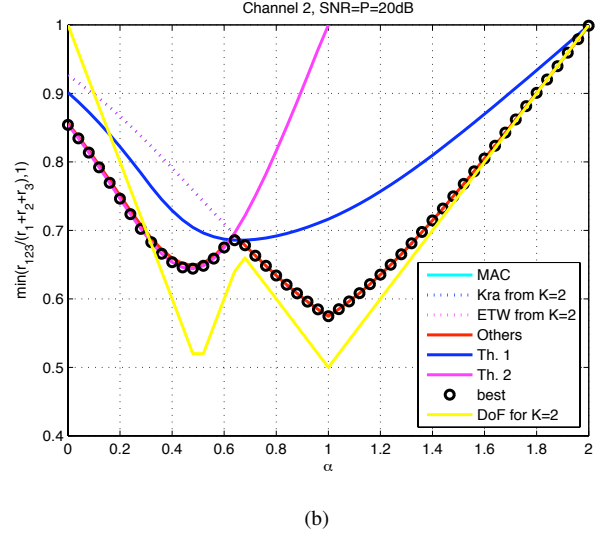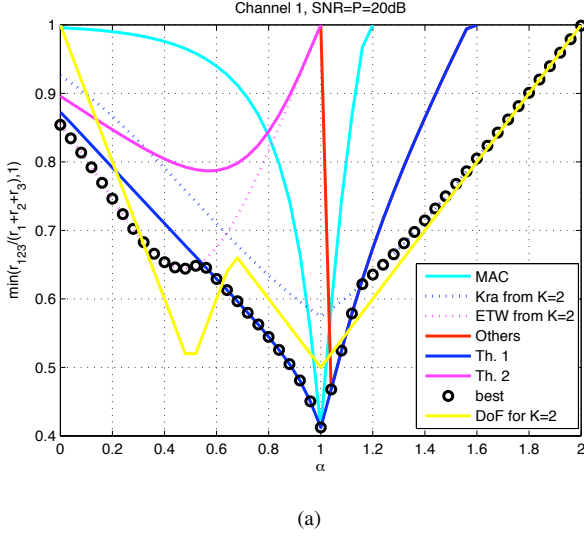
Fig. 3. $\mathrm{DoF}(P) \triangleq \min\{r_{123}/(r_1 + r_2 + r_3), 1\}$ vs. $\alpha = \log(P|h|^2)/\log(P)$ at $P = 20$dB for (a) the "fully symmetric" channel, and (b) the "cyclic symmetric" channel.

to the performance of the genie aided bound in [27], which is valid for $\alpha \geq 1$ only and coincides with $r_{123}^{(\mathrm{Th.1})}$.

The $\mathrm{DoF}(P)$ of the "cyclic symmetric" channel is presented in Fig. 3(b). From [29], $\mathrm{DoF} = \lim_{P \to \infty} \mathrm{DoF}(P)$ is as for the 2-user channel [21] (in this case there are no discontinuity since the channel matrix is always full rank). From Fig. 3(b) we see that the DoF (yellow line) does not accurately predict performance at $P = 20$dB yet. In this case, the bound $r_{123}^{(\mathrm{Th.2})}$ is the tightest for $\alpha \leq 2/3$ and $r_{123}^{(\mathrm{Kra})} = r_{123}^{(\mathrm{ETW})}$ (dotted magenta and blue lines) is the tightest and for $\alpha \geq 2/3$. The line labeled with "Others" (red line) refers to the outer bound in [29]; the bound $r_{123}^{(\mathrm{Th.2})}$ is tighter than [29] because of the optimized correlation coefficient between the noise of the side information signal and the noise of the channel output used in $r_{123}^{(\mathrm{Th.2})}$, as opposed to independent noises as used in [29]; as remarked in [22], at high SNR the optimal correlation is zero.

The performance of the "mixed strong-very strong" channel is not shown here because we know from [31] that $r_{123}^{(\mathrm{Kra})}$ is optimal.

IV. CONCLUSIONS

In this work we developed a framework to derive sum-rate upper bounds for the general memoryless interference channel with three source-destination pairs. For the Gaussian channel, the proposed bound is the tightest known for certain channel parameters.

ACKNOWLEDGMENT

APPENDIX

Lemma 5. *For two MISO AWGN channels*

$$Y_c = \boldsymbol{c}_c^H \boldsymbol{X} + Z_c, \ c \in \{1, 2\},$$

where $\boldsymbol{X} \sim \mathcal{N}(0, \boldsymbol{I})$ is independent of $Z_c \sim \mathcal{N}(0, 1)$, $c \in \{1, 2\}$, and $\boldsymbol{c}$ is a column vector of the same dimension of the input $\boldsymbol{X}$, we have

$$\min_{\rho \triangleq \mathbb{E}[Z_1 Z_2^*]: \ |\rho| \leq 1} \left\{ I(Y_1; \boldsymbol{X}|Y_2) \right\}$$
$$= \log\left(1 + |\boldsymbol{c}_1^H \boldsymbol{c}_2|(t + \sqrt{t^2 - 1})\right) - \log(1 + \|\boldsymbol{c}_2\|^2),$$

with

$$t \triangleq \frac{(1 + \|\boldsymbol{c}_1\|^2)(1 + \|\boldsymbol{c}_2\|^2) - |\boldsymbol{c}_1^H \boldsymbol{c}_2|^2 - 1}{2|\boldsymbol{c}_1^H \boldsymbol{c}_2|} \geq 1,$$

and where the minimum is attained by:

$$\rho^{(\mathrm{opt})} \triangleq (t - \sqrt{t^2 - 1}) e^{\mathrm{j} \angle \boldsymbol{c}_1^H \boldsymbol{c}_2} \in [0, 1] \quad \forall t \geq 1.$$

Proof: Assume $\boldsymbol{c}_1^H \boldsymbol{c}_2 \neq 0$. We have:

$$e^{I(Y_1; \boldsymbol{X}|Y_2)} = \frac{1 + \|\boldsymbol{c}_1\|^2 - \frac{|\boldsymbol{c}_1^H \boldsymbol{c}_2 + \rho|^2}{1 + \|\boldsymbol{c}_2\|^2}}{1 - |\rho|^2}$$

$$\geq \frac{(1 + \|\boldsymbol{c}_1\|^2)(1 + \|\boldsymbol{c}_2\|^2) - (|\boldsymbol{c}_1^H \boldsymbol{c}_2| + |\rho|)^2}{1 - |\rho|^2} \frac{1}{1 + \|\boldsymbol{c}_2\|^2}$$

$$\geq \left(1 + 2|\boldsymbol{c}_1^H \boldsymbol{c}_2| \min_{\rho: |\rho| \leq 1} \frac{t - |\rho|}{1 - |\rho|^2}\right) \frac{1}{1 + \|\boldsymbol{c}_2\|^2}$$

$$= \left(1 + \frac{|\boldsymbol{c}_1^H \boldsymbol{c}_2|}{t - \sqrt{t^2 - 1}}\right) \frac{1}{1 + \|\boldsymbol{c}_2\|^2}$$

$$= \left(1 + |\boldsymbol{c}_1^H \boldsymbol{c}_2|(t + \sqrt{t^2 - 1})\right) \frac{1}{1 + \|\boldsymbol{c}_2\|^2},$$

as claimed. ∎

Remark 3. *The function* $\min_\rho\{I(Y_1; \boldsymbol{X}|Y_2)\}$ *is decreasing in the angle between the vectors* $\boldsymbol{c}_1$ *and* $\boldsymbol{c}_2$*. For* $|\boldsymbol{c}_1^H \boldsymbol{c}_2|^2 = \|\boldsymbol{c}_1\|^2 \|\boldsymbol{c}_2\|^2$ *(i.e., parallel channel vectors), we have:*

$$\min_\rho I(Y_1; \boldsymbol{X}|Y_2) = \log\left(\frac{1 + \max\{\|\boldsymbol{c}_1\|^2, \|\boldsymbol{c}_2\|^2\}}{1 + \|\boldsymbol{c}_2\|^2}\right)$$

as for SISO (degarded) channels. For $|\boldsymbol{c}_1^H \boldsymbol{c}_2| = 0$ *(i.e., orthogonal channel vectors), one can easily see that* $|\rho| = 0$ *is optimal and thus:*

$$\min_\rho I(Y_1; \boldsymbol{X}|Y_2) = \log(1 + \|\boldsymbol{c}_1\|^2).$$

Remark 4. *By using* $\rho = 0$ *we would get:*

$$I(Y_1; \boldsymbol{X}|Y_2)|_{\rho=0} = \log\left(1 + \|\boldsymbol{r}_1\|^2 - \frac{|\boldsymbol{c}_1^H \boldsymbol{c}_2|^2}{1 + \|\boldsymbol{r}_2\|^2}\right)$$
$$\in \left[\log\left(1 + \frac{\|\boldsymbol{r}_1\|^2}{1 + \|\boldsymbol{r}_2\|^2}\right), \ \log\left(1 + \|\boldsymbol{r}_1\|^2\right)\right].$$

REFERENCES

[1] D. Tuninetti, "An Outer Bound Region for the K-user Interference Channel," *Proceedings 2011 IEEE International Symposium on Information Theory*, July 2011.

[2] A. El Gamal and Y.-H. Kim, "Lecture notes on network information theory," *submitted, preprint at arXiv:1001.3404*, 2010.

[3] H. Sato, "On the capacity region of a discrete two-user channel for strong interference," in *IEEE Trans. Inform. Theory*, vol. 24(3), May 1978, pp. 377–379.

[4] M. H. M. Costa and A. A. E. Gamal, "The capacity region of the discrete memoryless interference channel with strong interference," in *IEEE Trans. Inform. Theory*, vol. 33(5), Sept 1987, pp. 710–711.

[5] A. B. Carleial, "A case where interference does not reduce capacity," in *IEEE Trans. Inform. Theory*, vol. 21(5), Sept 1975, pp. 569–570.

[6] A. A. E. Gamal and M. H. M. Costa, "The capacity region of a class of deterministic interference channels," in *IEEE Trans. Inform. Theory*, vol. 28(2), March 1982, pp. 343–346.

[7] N. Liu and S. Ulukus, "The capacity region of a class of discrete degraded interference channels," in *IEEE Trans. Inform. Theory*, vol. 54(9), Sept 2008, pp. 4372–4378.

[8] R. Benzel, "The capacity region of a class of discrete additive degraded interference channels," *IEEE Trans. Inform. Theory*, vol. IT-25, pp. 228–231, Mar 1979.

[9] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," in *IEEE Trans. Inform. Theory*, vol. 27(1), Jan 1981, pp. 49 –60.

[10] H. Sato, "Two-user communication channels," in *IEEE Trans. Inform. Theory*, vol. 23(3), 1977, pp. 295 – 304.

[11] A. B. Carleial, "Outer bounds on the capacity of interference channels," in *IEEE Trans. Inform. Theory*, vol. 29, no. 4, July 1983, pp. 602–606.

[12] G. Kramer, "Outer bounds on the capacity of gaussian interference channels," in *IEEE Trans. Inform. Theory*, vol. 50, no. 3, Jan 2004, pp. 581–586.

[13] H. Sato, "The capacity of the gaussian interference channel under strong interference," in *IEEE Trans. Inform. Theory*, vol. 27(6), Nov 1981, pp. 786–788.

[14] M. H. M. Costa, "On the gaussian interference channel," in *IEEE Trans. Inform. Theory*, vol. 31(5), Sept 1985, pp. 607–615.

[15] D. Tuninetti and Y. Weng, "On gaussian mixed interference channels," in *IEEE International Symposium Information Theory*, Toronto, Canada, July 2008.

[16] A. S. Motahari and A. K. Khandani, "Capacity bounds for the gaussian interference channel," *IEEE Trans. Inform. Theory*, vol. 55, no. 2, pp. 620–643, 2009.

[17] I. Sason, "On achievable rate regions for the gaussian interference channel," in *IEEE Trans. Inform. Theory*, vol. 50(6), June 2004, pp. 1345–1356.

[18] X. Shang, G. Kramer, and B. Chen;, "A new outer bound and the noisy-interference sumrate capacity for gaussian interference channels," in *IEEE Trans. Inform. Theory*, vol. 55(2), Feb 2009, pp. 689 – 699.

[19] V. S. Annapureddy and V. V. Veeravalli, "Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region," in *IEEE Trans. Inform. Theory*, vol. 55(7), July 2009, pp. 3032 – 3050.

[20] P. Razaghi and G. Caire, "On achievability of gaussian interference channel capacity to within one bit," *submitted to IEEE Transactions on Information Theory, Dec 2010, arXiv:1101.2483*, 2010.

[21] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inform. Theory*, vol. 54, no. 12, pp. 5534 – 5562, 2008.

[22] R. Etkin, "New sum-rate upper bound for the two-user gaussian interference channel," *Information Theory, 2009. ISIT 2009. IEEE International Symposium on*, pp. 2582 – 2586, 2009.

[23] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom for the k user interference channel," *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug 2008.

[24] B. Nazer and M. Gastpar, "Computation over multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3498–3516, Oct. 2007.

[25] G. Bresler, A. Parekh, , and D. Tse, "The approximate capacity of the many-to-one and one-to-many gaussian interference channel, arxiv:0809.3554," *Proceedings of the Allerton Conference*, Sept 2007.

[26] S. Sridharan, A. Jafarian, S. Vishwanath, S. Jafar, and S. Shamai, "A layered lattice coding scheme for a class of three user gaussan interference channels," *Communication, Control, and Computing, 2008 46th Annual Allerton Conference on*, pp. 531 – 538, Sept 2008.

[27] V. R. Cadambe and S. A. Jafar, "Parallel gaussian interference channels are not always separable," *IEEE Trans. Inform. Theory*, vol. 55(9), pp. 3983–3990, Sept 2009.

[28] J. Jose and S. Vishwanath, "Sum capacity of k user gaussian degraded interference channels," *Proc. IEEE Info. Theory Workshop (ITW), Dublin, Ireland.*, Sept 2010.

[29] L. Zhou and W. Yu, "On the symmetric capacity of the k-user symmetric cyclic gaussian interference channel," *Proceedings of Conference on Information Science and Systems (CISS), Princeton, NJ*, March 2010.

[30] B. Bandemer, G. Vazquez-Vilar, and A. E. Gamal, "On the sum capacity of a class of cyclically symmetric deterministic interference channels," *Proc. IEEE International Symposium on Information Theory, Seoul, Korea*, p. 26222626, July 2009.

[31] A. Chaaban and A. Sezgin, "The capacity region of the 3-user gaussian interference channel with mixed strong-very strong interference, arxiv:1010.4911," *Submitted to ISIT 2011*, 2010.

[32] S. Sridharan, A. Jafarian, S. Vishwanath, and S. A. Jafar, "Capacity of symmetric k-user gaussian very strong interference channels," *Globecom 2008*, 2008.

[33] S. A. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric gaussian k user interference channel," *IEEE Trans. Inform. Theory*, vol. 56(7), pp. 3297–3303, July 2010.

[34] R. Etkin and E. Ordentlich, "On the degrees-of-freedom of the k-user gaussian interference channel," *Information Theory, 2009. ISIT 2009. IEEE International Symposium on*, pp. 1919 – 1923, 2009.

[35] S. H., "An outer bound to the capacity region of broadcast channels," *IEEE Trans. Inform. Theory*, pp. 374 – 377, May 1978.

[36] S. E. Yang and D. Tuninetti, "On interference channels with generalized feedback (a.k.a. with source cooperation). part i achievable region, in print, arxiv:1006.1667," in *IEEE Trans. Inform. Theory*, June 2010.

[37] S. Yang and D. Tuninetti, "A new sum-rate outer bound for gaussian interference channels with generalized feedback," in *Proceedings of 2009 IEEE International Symposium on Information Theory (ISIT 2009), Seoul, South Korea*, June 2009.

[38] D. Tuninetti, "An outer bound region for interference channels with generalized feedback," *Proceedings of the Information Theory and Applications Workshop, 2010, San Diego, CA USA*, Jan 2010.

[39] S. Rini, D. Tuninetti, and N. Devroye, "New inner and outer bounds for the discrete memoryless cognitive channel and some capacity results," *Information Theory, IEEE Transactions on*, 2010, in print, preprint at arXiv:1003.4328.

[40] ——, "Outer bounds for the interference channel with a cognitive relay," *Proc. IEEE Information Theory Workshop (ITW), Dublin, Ireland (preprint at arXiv:1004.4944)*, Sep 2010.

[41] V. Prabhakaran and P. Viswanath, "Interference channels with source cooperation," *submitted to IEEE Trans. Info. Theory in May 2009, Arxiv preprint arXiv:0905.3109v1*, 2009.

[42] H. V. Poor, *An Introduction to Signal Detection and Estimation, 2nd ed.* New York: Springer-Verlag, 1994.